Monotonic Inferences under Epistemic Verbs

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January Project - *Team Semantics and Dependence: Linguistic and Philosophical Applications*

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Outline

Introduction

A puzzle about monotonicity under knowledge

Quantified bilateral state-based epistemic logic (QBSEL)

Monotonic inferences

Monotonicity is a general concept that describes ‘order preserving’ properties of functions over partially ordered domains.

In natural language, monotonicity is reflected in the semantic properties of determiners. Given a non-empty domain D, a determiner over D is a function: $\mathcal{P}(D) \times \mathcal{P}(D) \to \{0, 1\}$. Hence, a determiner is a relation between subsets of D.

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Definition (Upward monotone determiner)

A determiner Det of type $\langle 1, 1 \rangle$ is *upward monotone* in the first (second) argument iff if $Det(A_1, A_2)$ and $A_1 \subseteq B$, then $Det(B, A_2)$

Monotonic inferences

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E.g. $\downarrow Every \uparrow$, $\uparrow Some \uparrow$

- a. Every **student** in the school is running ⇒ Every **female student** in the school is running (downward)

  b. Every reading room is **equipped with a Mac computer** ⇒ Every reading room is **equipped with a computer** (upward)

- a. Some **dogs** bark ⇒ Some **animals** bark (upward)

  b. Some dogs are **barking loudly** ⇒ Some dogs are **barking** (upward)

Monotonicity under modals

The monotonicity properties for the determiners will be reversed, when the quantified sentences are embedded under negation:

- a. Not every **student** in the school are running. ⇒ Not every **people** in the school are running (upward)

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- a. Nicholas wants to get a free trip on the Concorde.
  ?⇒ b. Nicholas wants to get a trip on the Concorde.
  ([Asher,1987], under desire)

Classical semantics of epistemic verb ‘know’

According to canonical interpretation of epistemic modalities in Hintikka-style, “know” behaves like the necessity modality $\Box$. So if an agent a knows $\phi \rightarrow \psi$ and ϕ, we can conclude a knows ψ. Monotonicity is not blocked by the classical semantics of ‘know’.

- a. Tom knows Susan is a wealthy lady
  $\Rightarrow$ b. Tom knows Susan is a lady

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- a. Tom knows Susan is a wealthy lady
  $\Rightarrow$ b. Tom knows Susan is a lady

If Tom wants to marry a wealthy lady, will Tom marry Susan?

The puzzle

Tom is the organiser of a pet party at ILLC, which is open to all kinds of pets. He thinks Susan may need to be informed that cats are at this party as she is allergic to cats. Meanwhile the drinks for the party have not yet arrived so he needs to pick them up. For some reason Tom can't do both at the same time, and he has to decide which one to do. Tom is torn at the moment because either Susan has a bad allergic reaction from the party or the lack of drinks will make the party fail.

Just then Tom hears two colleagues A and B in the common room chatting about Susan, and A says:

(1) *Susan knows that some animals will be at the party.*

Based on this piece of information Tom concludes that it is not necessary to warn Susan.

The puzzle

Eventually, however, Susan comes to the party and gets a serious allergic reaction. Tom blames A and asks furiously "Why did you say that Susan knew that there would be animals at the party?" And A replies:

(2) *What I said was true, Susan knew that animals would be at the party, because she knew there would be dogs (she saw some other colleague preparing her dog and thought that it was a dog party). So by monotonicity, I reasoned:*

a. *Susan knows that some dogs will be at the party.*
b. *Susan knows that dogs are animals.*
c. *Therefore Susan knows that some animals will be at the party.*

The puzzle

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It seems that from (1) Tom concluded that

(3) Susan knows that there might be some cats at the party.

Disjunctive meaning of a predicate

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(4) a. Jack bought a Porsche or a Ferrari.

$\rightsquigarrow$ b. The speaker does not know which car Jack actually bought. It might be a Porsche and it might be a Ferrari.

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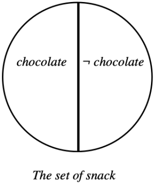

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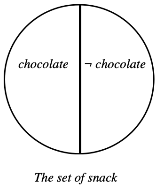

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When a predicate Q which semantically includes P ($P \subset Q$) is uttered, Q will be understood with respect to a partition provided by P and its negation. We will call P the *sub-predicate* of Q.

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Susan knows that some cats might be at the party.

Example 2

Consider the following example:

> *Assume the same scenario as in Example 1 and further suppose that the two colleagues A and B in the common room were having a bet on whether Susan would answer correctly to the following question:*
>
> (5) *Would there be animals at the ILLC party?*

In the described scenario it would be perfectly rational for A to bet that Susan would answer yes according to the reasoning process from (2)a to (2)c.

Impact of context

By this example, we argue that the monotonic reasoning is useful in some cases.
In Example 2 the issue under discussion is “whether Susan knows that some animals will be at the party”, a question about the predicate “animals”. The information from (2)a to (2)c is sufficient for the colleague A to make the right decision, because the conclusion appropriately settles the issues under discussion. In contrast in Example 1 the central issue is “whether Susan knows that some cats will be at the party” which makes the predicate “cats” salient, rather than “animals”. So statement (1), which is derived from (2) communicating information about animals, cannot support Tom in making correct predictions on knowledge about cats. It is only in this latter case that the loss of information caused by the monotonicity step becomes problematic.

Syntax

Term $t ::= c \mid x$

Formula

$\varphi ::= P^n t_1 ... t_n \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \exists x \varphi \mid K_a \varphi \mid \Diamond \varphi \mid NE$

Semantics

- An epistemic model for $\mathcal{L}_{\mathcal{E}}$ is a tuple $\mathcal{M} =< W, D, R, I >$. R is an equivalence relation on W.

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$K_S \exists x[(Cx \wedge P) \vee (\neg Cx \wedge P)]^+ \vDash K_S \Diamond \exists x \ (Cx \wedge Px) \wedge K_S \Diamond$ $\exists x(\neg Cx \wedge Px)$

Gains and losses

In QBSML, the epistemic possibility is defined with respect to the relation R, but the following fact cannot be proved.

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By the non-relation semantics of 'might' in QBSEL, we can prove the above fact, but the problem arise when the operator K interacts with $\Diamond$, i.e., the following reasoning is **invalid** in QBSEL:

- $K \Diamond \phi \vDash \Diamond \phi$

Thank You!!!